Square Root of a Function

Graphing $y = f(x)$ and $y = \sqrt{f(x)}$

- To graph $y = \sqrt{f(x)}$, you can set up a table of values for the graph of y = f(x). Then, take the square root of the elements in the range, while keeping the elements in the domain the same. The mapping of this transformation would be $(x, y) \to (x, \sqrt{y})$.

- When graphing $y = \sqrt{f(x)}$, pay attention to the invariant points, which are points that are the same for $y = f(x)$ as they are for $y = \sqrt{f(x)}$. The invariant points are (x, 0) and (x, 1) because when $f(x) = 0$, $\sqrt{f(x)} = 0$, and when $f(x) = 1$, $\sqrt{f(x)} = 1$

Domain and Range of $y = \sqrt{f(x)}$

- The domain of $y = \sqrt{f(x)}$ is any value of x for which $f(x) \geq 0$ (since you cannot take the square root of a negative number).

- The range is the square root of any value in $y = f(x)$ for which $y = \sqrt{f(x)}$ is defined.

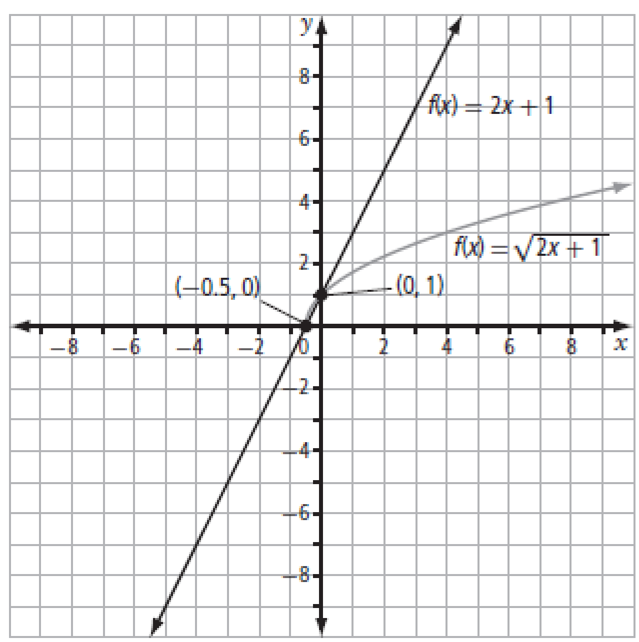
The Graph of $y = \sqrt{f(x)}$

Value of f(x)	$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
Location of $y = \sqrt{f(x)}$ relative to $y = f(x)$	The graph of $y = \sqrt{f(x)}$ is *undefined.*	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ *intersect* on the x-axis.	The graph of $y = \sqrt{f(x)}$ is ***above*** the graph of $y = f(x)$.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ ***intersect.***	The graph of $y = \sqrt{f(x)}$ is ***below*** the graph of $y = f(x)$.

Example 1: Compare Graphs of a Linear Function and the Square Root of the Function

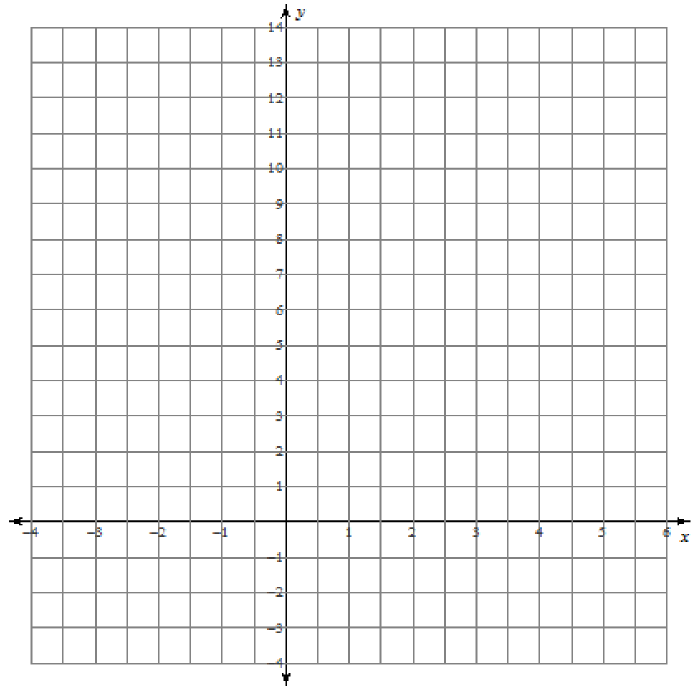
Consider the graphs of $f(x) = 2x + 1$ and $f(x) = \sqrt{2x+1}$ shown to the right. Note that the graph of $f(x) = \sqrt{2x+1}$ is undefined for _________. The graphs of $f(x) = 2x + 1$ and $f(x) = \sqrt{2x+1}$ intersect at _________ and at _________. The graph of $f(x) = \sqrt{2x+1}$ is *above* the graph of $f(x) = 2x + 1$ for ________________, and *below* the graph of $f(x) = 2x + 1$ for _________.

(Why?)



Example 2: Compare Graphs of a Linear Function and the Square Root of the Function

a. Given $f(x) = 4x - 3$, graph the functions $y = f(x)$ and $y = \sqrt{f(x)}$.
b. Compare the graphs.



Solution:

a.

x	$f(x) = 4x - 3$	$f(x) = \sqrt{4x - 3}$
0		
0.75		
0.77		
0.8		
1		
2		
3		
4		

b. **Comparison:**

The graphs of $f(x) = 4x - 3$ and $f(x) = \sqrt{4x - 3}$ intersect at __________ and at __________.
These are referred to as _________________ points.

The x-intercept of the graph of $f(x) = 4x - 3$ is also the x-intercept *and* the _____________ point of the graph of the $f(x) = \sqrt{4x - 3}$.

The graph of $f(x) = \sqrt{4x - 3}$ is *above* the graph of $f(x) = 4x - 3$ for ____________________.

The graph of $f(x) = \sqrt{4x - 3}$ is *below* the graph of $f(x) = 4x - 3$ for _______________.

Domain of $f(x) = 4x - 3$: ___________________

Range of $f(x) = 4x - 3$: _____________________

Domain of $f(x) = \sqrt{4x - 3}$: ____________________

Range of $f(x) = \sqrt{4x - 3}$: _____________________

Example 3: Graph the Square Root of a Function from the Graph of the Function and Explore the Domains and Ranges

For each of the following functions, y = f(x), sketch the graph of $y = \sqrt{f(x)}$. Determine the domain and range of y=f(x) and $y = \sqrt{f(x)}$.

a. $y = -x - 2$ and $y = \sqrt{-x - 2}$

b. $y = x^2 - 6x + 13$ and $y = \sqrt{x^2 - 6x + 13}$

c. $y = (x+3)^2 - 4$ and $y = \sqrt{(x+3)^2 - 4}$

d. $y = 9 - x^2$ and $y = \sqrt{9 - x^2}$

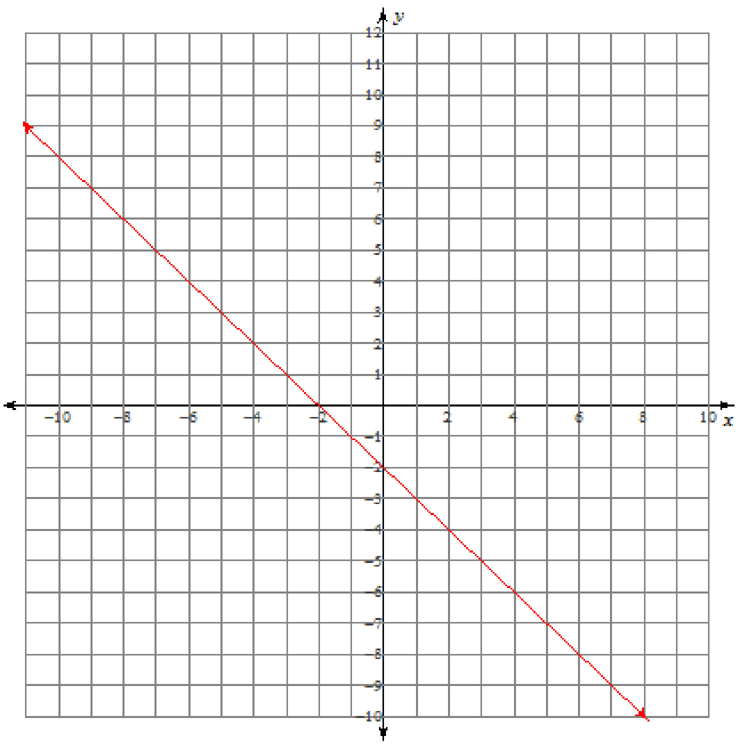
e. $y = -x^2 - 1$ and $y = \sqrt{-x^2 - 1}$

Solution:

On the same grid as the graph of y = f(x):

- Plot the invariant points.
- Draw a smooth curve between the invariant points, and *above* the graph of y = f(x).
- Plot a few other points. If possible, choose values of f(x) which have simple square roots.

a. $y = -x - 2$ and $y = \sqrt{-x - 2}$



$y = -x - 2$:

Domain: ______________________________

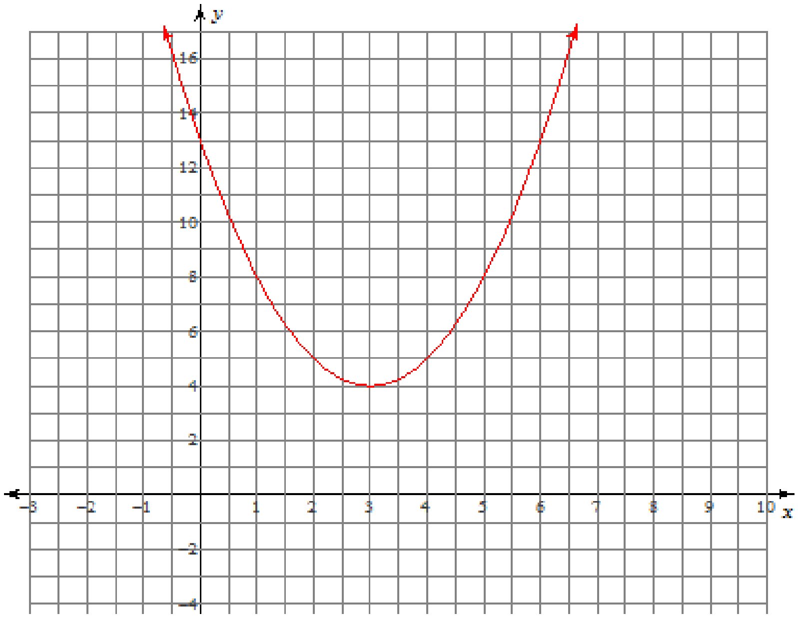
Range: ______________________________

$y = \sqrt{-x - 2}$:

Domain: ______________________________

Range: ______________________________

b. $y = x^2 - 6x + 13$ and $y = \sqrt{x^2 - 6x + 13}$



$y = x^2 - 6x + 13$:

Domain: ____________________

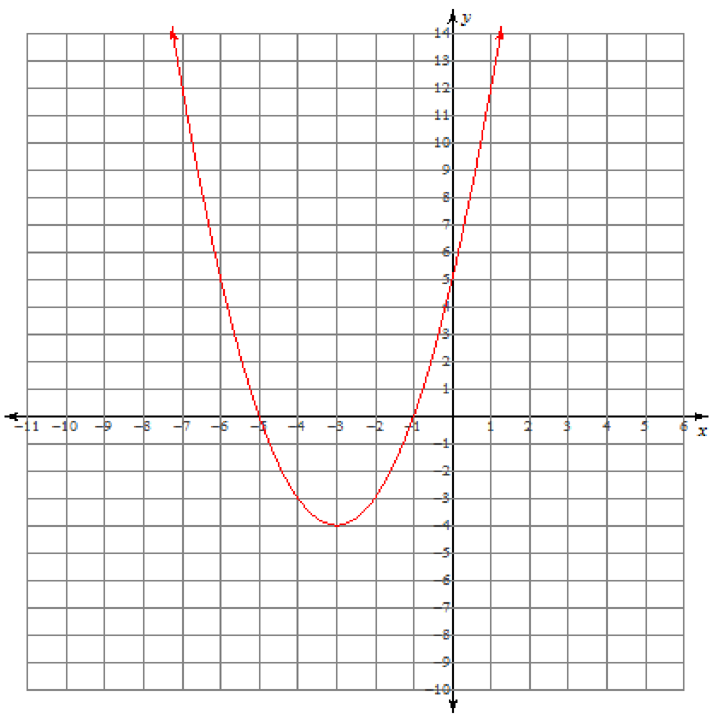
Range: ____________________

$y = \sqrt{x^2 - 6x + 13}$:

Domain: ____________________

Range: ____________________

c. $y = (x+3)^2 - 4$ and $y = \sqrt{(x+3)^2 - 4}$



$y = (x+3)^2 - 4$:

Domain: ____________________

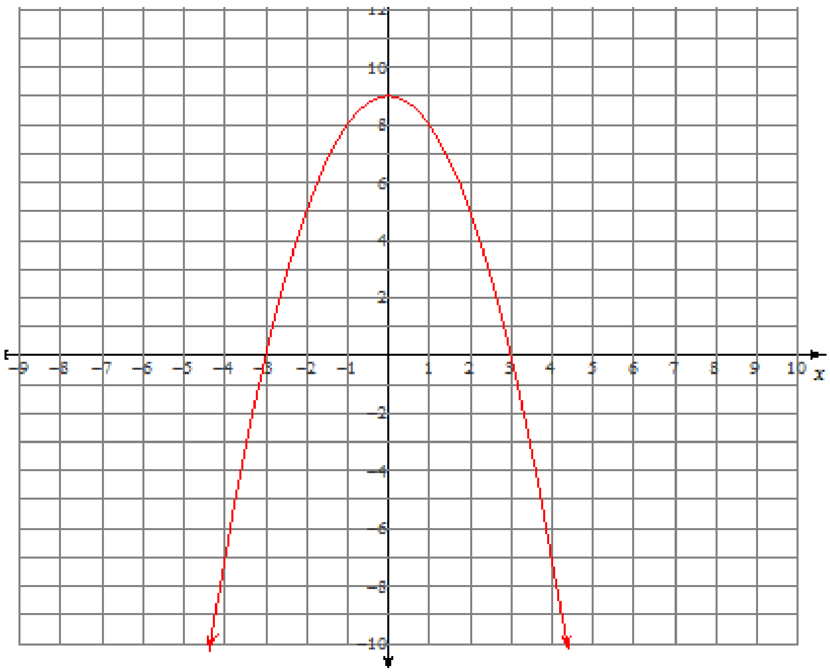
Range: ____________________

$y = \sqrt{(x+3)^2 - 4}$:

Domain: ____________________

Range: ____________________

d. $y = 9 - x^2$ and $y = \sqrt{9 - x^2}$



$y = 9 - x^2$:

Domain: ______________________

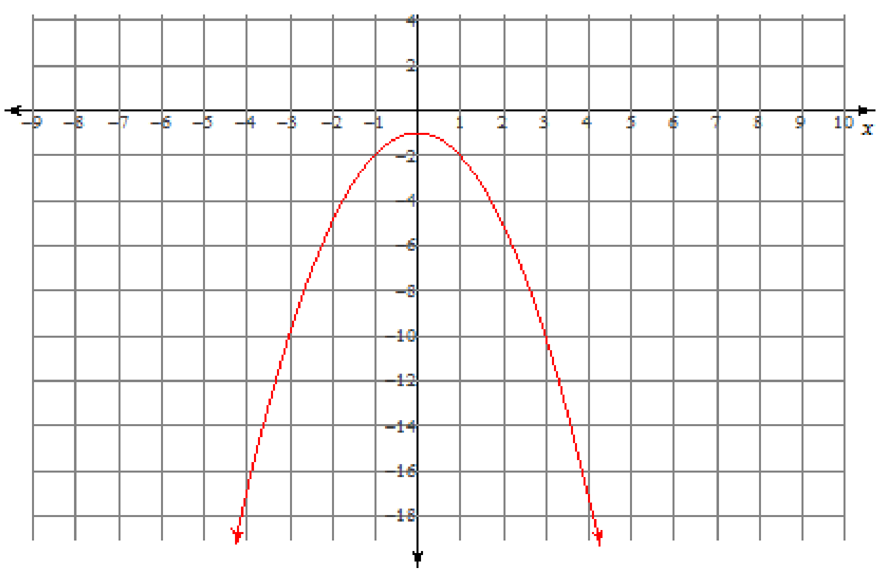
Range: ______________________

$y = \sqrt{9 - x^2}$:

Domain: ______________________

Range: ______________________

e. $y = -x^2 - 1$ and $y = \sqrt{-x^2 - 1}$



$y = -x^2 - 1$:

Domain: ______________________

Range: ______________________

$y = \sqrt{-x^2 - 1}$:

Domain : ______________________

Range : ______________________

Example 4: Determine Domains and Ranges of $y = f(x)$ and $y = \sqrt{f(x)}$

For the functions in each pair :

i) Determine the coordinates of the invariant points.

ii) Sketch the graph of $y = f(x)$ using key points (invariant points, vertex, etc.) and use this graph to help you sketch the graph of $y = \sqrt{f(x)}$.

iii) Determine the domains and ranges of the functions.

a. $y = 4x - 2$, $y = \sqrt{4x-2}$ b. $y = 12 - 3x^2$, $y = \sqrt{12-3x^2}$ c. $y = 0.5x^2 - 5$, $y = \sqrt{0.5x^2 - 5}$

Solution:

a. $y = 4x - 2$ and $y = \sqrt{4x-2}$

i) Invariant points:

ii) Sketches:

iii) Domains and Ranges:

Domain of $y = 4x - 2$: ______________________ Range of $y = 4x - 2$: ______________________

Domain of $y = \sqrt{4x-2}$: ____________________ Range of $y = \sqrt{4x-2}$: ____________________

b. $y = 12 - 3x^2$ and $y = \sqrt{12 - 3x^2}$

i) Invariant points:

ii) Sketches:

iii) Domain of $y = 12 - 3x^2$: ______________________ Range of $y = 12 - 3x^2$: ______________________

Domain of $y = \sqrt{12 - 3x^2}$: ______________________ Range of $y = \sqrt{12 - 3x^2}$: ______________________

c. $y = 0.5x^2 - 5$ and $y = \sqrt{0.5x^2 - 5}$

i) Invariant points:

ii) Sketches:

iii) Domain of $y = 0.5x^2 - 5$: ______________________ Range of $y = 0.5x^2 - 5$: ______________________

Domain of $y = \sqrt{0.5x^2 - 5}$: ______________________ Range of $y = \sqrt{0.5x^2 - 5}$: ______________________